Supply Chain Dynamics With Assortative Matching

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Abstract

This paper studies the evolutionarily stable strategies of one-manufacturer and one-retailer supply chains. Each manufacturer and retailer chooses between two pure strategies of management: shareholder-oriented or stakeholder-oriented. Based on its management strategy, the firm decides its wholesale or retail price. In this paper, we consider supply chains formed by two matching processes: random matching and assortative matching. Our results indicate that random matching does not support interior Nash equilibria; the evolutionarily stable strategy is for both manufacturer and retailer to choose shareholder strategy. We extend Bergstrom (2003) to a two-population game, and compare the dynamics of supply chains under random matching and assortative matching. Interior Nash equilibrium is observed with assortative matching. However, this interior equilibrium is unstable. The four unique strategy profiles obtained by various combinations of the two strategy choices may be evolutionarily stable for certain values of the indices of assortativity.

Keywords: Nash equilibrium; assortative matching; evolutionary stable strategy; replicator equation

JEL Classification: C73; L21; D21

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1 Introduction

Random matching is a popular matching pattern in economic theory. In random matching, the probability of matching with an agent of a particular type is independent of one's own type. However, most real-life matches, like education, marriage, employment, or decision about where to live are not results of random matching. Various types of non-random matching have been identified; the most common of which are positive and negative assortment (Thiessen and Gregg, 1980). Assortative matching was introduced by Becker (1973) in his study of the marriage market. In contrast to random matching, the probability of various matches is determined by agents' types in assortative matching. Thus, based on the relevant matching rule, agents of the same type may be more or less likely to match than would be expected under random matching. With positive assortative matching, agents with the same strategy choice are more likely to match than would be expected with random matching. With negative assortative matching, the correlation between strategy profiles of partners of a match is negative.

Examples of assortative matching abound in the natural world. Land snails are disproportionately likely to pair up with others of similar size. Among bluebirds, more brightly colored males mate with more brightly colored females and less brightly colored birds tend to pair together (Olsson, 1993; McMillan et al., 1999). Supply chains also exhibit assortative matching. Machikita and Ueki (2012) provide evidence for strong assortative matching by firm attributes, like multinational enterprise, joint venture, and local firm. Costinot et al. (2013) exploit the tendency of positive assortative matching between countries' productivities and assignment to stages of production in developing a model of global supply chains. This paper studies assortative matching between manufacturers and retailers in single-manufacturer and single-retailer supply chains. Every manufacturing and retailing firm chooses between two strategy profiles; they can be either shareholder-oriented or stakeholder-oriented. The objective of a shareholder-oriented firm is to maximize shareholder payoff. On the other hand, a stakeholder-oriented firm seeks to maximize the utility of all stakeholders, such as suppliers, retailers, customers etc., and not just the shareholder. Firms choose their product price based on their management strategy.

In Section 3, we introduce our basic supply chain model and present results of a one-shot game. We show that firms' choice of price is affected by how stakeholder-oriented (on the intensive and the extensive margin) their partner in the supply chain. In Section 4, we analyze the evolutionary stability of random matching equilibria. There is no interior Nash equilibrium in the game with random matching and that the evolutionarily stable strategy (ESS) is for both manufacturer and the retailer to choose shareholder-oriented strategy. In Section 5, we use the index of assortativity to analyze evolutionary stability of a game of assortative matching. We contrast dynamics of the game under the two matching rules. With assortative matching, the interior equilibrium is not asymptotically stable and all four possible strategy profiles can be supported as ESS for different values of the indices of assortativity.

2 Literature Review

In the stakeholder model of corporation, firms' operations are guided by a broad sense of responsibility to its stakeholders, and not just shareholders. Stakeholders of a firm can include employees, suppliers, distributors, consumers, etc. Lusch and Laczniak (1987) shows that a business system with firms that want to maximise stakeholder utility could have more long run stability and evolutionary potential. Harrison et al. (2010) describe how and why stakeholder-oriented management leads to sustainable competitive advantage in some firms. Calabrese et al. (2013) point out that considering stakeholder needs can be an opportunity rather than a constraint, and, by meeting them, companies may establish a basis for competitive advantages. Our work is related to the growing literature on the role of various stakeholder groups in firms; see, for example, Blinder (1998), Pagano and Volpin (2005), Lee (2008), Bhattacharya and Korschun (2008), Pascoe et al. (2009), and Boesso and Kumar (2009). In this paper, we consider manufacturers and retailers as each other's stakeholder.

We also contribute to the literature on the dynamics of a supply chain with individual preferences. Previous research shows that a supply chain can be coordinated with preference for social responsibility (Amaeshi et al., 2008; Panda, 2014; Goering, 2012; Hua and Li, 2008; Hsueh, 2014; Ni et al., 2010; Lau and Lau, 2002). Xiao and Yu (2006) and Xiao and Chen (2009) study evolutionarily stable strategies when retailers choose between profit maximising and revenue maximising strategies. They find that four different strategy profiles may be evolutionarily stable under various conditions in a Cournot market. Chai et al. (2015) discuss evolutionary dynamics when firms choose to be either stakeholder-oriented or shareholder-oriented and analyse the evolution of firms' preference parameter. These analyses consider games of random matching. However, as Friedman and Sinervo (2016) point out, there are many other kinds of social structures that modulate strategic interactions, and our paper considers assortative matching in supply chains.

Assortative matching is a nonrandom matching pattern in which individuals of the same type match with each other more or less frequently than would be expected under a random matching pattern. Shimer and Smith (2000) and Siow et al. (2009) study the characteristics of assortative matching based on the results of Becker (1973). Legros and Newman (2007), and Schulhofer-Wohl (2006) study the dynamics of assortative matching in an economy where utility is not transferable between partners. Durlauf and Seshadri (2003) study the conditions under which assortative matching is efficient and show that characterizing cross-section evolution of an efficiently sorted economy is likely to be highly complex. Others have considered assortative matching between firms and workers in labour markets (Andrews et al., 2008; Andersson et al., 2007; Mendes et al., 2010; Eeckhout and Kircher, 2012). We contribute to this discussion by studying how assortative matching affects firms' decisions to be shareholder-oriented or stakeholder-oriented.

We extend Bergstrom (2003) to a two-population game, and compare the dynamics of supply chains under random matching and assortative matching. To the best of our knowledge, our paper is the first to investigate the index of assortativity for two populations. Previous work exists on assortative matching with two populations. Shirata (2012) studies the evolution of fairness in an ultimatum game with noise in learning under assortative matching. Atakan (2006) extends the results of Becker (1973) to heterogeneous agents and derives that complementarities in joint production (supermodularity of the joint production function) lead to assortative matching. Hoppe et al. (2009) consider two-sided markets with a finite number of agents on each side, and with two-sided incomplete information. While these pervious papers study assortative matching with matching rules, our analysis is based on the index of assortativity. We find that the indices of assortativity determine whether the match between manufacturers and retailers is negative assortative or positive assortative.

Bergstrom (2013) shows the relationship between the index of assortativity and Wright's F-statistic for a two-pool assortative matching process. Here, a member of any given population matches with either a member of an assortative pool comprised only of its own type or with a member from a random pool comprised of all who did not match in the assortative pool. Our analysis compares two different populations (manufacturers and retailers), and further each population has two types of agents. Matches are made across the population. Alger and Weibull (2013), and Alger (2010) study the evolutionary stability of preferences using the index of assortativity. We analyse the evolutionary stable strategy in supply chains using the index of assortativity and the method of replicator dynamics.

3 The Model

Consider a market that consists of two large populations: manufacturers and retailers. Each manufacturer and retailer chooses between two strategies of management: shareholder oriented (H/h) or stakeholder oriented (T/t). We denote manufacturers' choices with uppercase letters and retailers' choices with lowercase letters. We assume that the number of manufacturers and retailers in the economy are equal to each other. This assumption makes the game symmetric and enables us to consider games where channel power is not skewed towards the group (manufacturer or retailer) with fewer members.[1]

The time sequence of the game is as follows. First, manufacturers produce homogenous final goods at marginal cost c. Based on their strategy choice, each manufacturer chooses the wholesale price (w) that their retailers will face. Second, manufacturers and retailers are matched with each other according to some known matching rule to form single-manufacturer and single-retailer supply chains. Third, the retailer chooses their market price (p) based on the retailer's strategy choice. The retailer faces the following demand function:

$$D = a - p \tag{1}$$

The parameter a expresses market potential. We assume that $a > c$ to ensure positive demand when retail price equals unit production cost. We also assume that for any given market price, the manufacturer faces inelastic demand; demand is 0 for all $w > (a-p)$ and demand is constant and determined by Equation (1) for all $w < (a-p)$.

Since the manufacturer and the retailer each face two strategy choices, there are four possible strategy profiles for any given supply chain: (i) both manufacturer and retailer are stakeholder-oriented (Tt); (ii) both manufacturer and retailer are shareholder-oriented (Hh); (iii) manufacturer is shareholder-oriented and retailer is stakeholder-oriented (Ht); or, (iv) manufacturer is stakeholder-oriented and retailer is shareholder-oriented (Th). We denote each strategy profile by a superscript; e.g., superscript Tt denotes case (i). We refer to Tt as symmetric stakeholder strategy, Hh as symmetric shareholder strategy, and Ht and Th as asymmetric strategy profiles.

The firms' value function is determined by its choice to be shareholder-oriented or stakeholder-oriented. A shareholder-oriented firm seeks to maximise its own profit, which will then be distributed to its exogenously given shareholders. We model

[1] Moreover, our assumption that populations are of the size affords us the benefit of efficiency when we consider a game of assortative matching (Durlauf and Seshadri, 2003).

shareholder firms' value functions with their profit functions. In our analysis, each firm's stakeholder is its partner in the supply chain. As is common in the literature, we model stakeholder firms' value functions by incorporating interests of all its stakeholders. Thus, a stakeholder-oriented manufacturers value function incorporates a scaled version of the retailer's profit function; and vice versa for a stakeholder-oriented retailer.

Next, we consider value functions of agents of both types in both the manufacturer and the retailer populations. Equations (2) and (3)are the value functions of stakeholder-oriented manufacturers and retailers respectively.

$$\max_{w}(w-c)(a-p)+k_m(p-w)(a-p), \tag{2}$$

$$\max_{p}(p-w)(a-p)+k_r(w-c)(a-p). \tag{3}$$

In both equations, the first term is the firm's profit function and the second term is its stakeholder's profit function. The parameter k_m denotes how much the retailer's profit factors into the manufacturer's value-optimising decision. Similarly, the parameter k_r denotes the weight attached by the retailer to its stakeholder's profit. In our analysis, these parameter values are population-specific and commonly known. For a stakeholder-oriented firm, $0 \leq k_m, k_r \leq 1$. Shareholder-oriented firms can be viewed as a special case of this characterisation, with k_m =0, and $k_r = 0$.

We characterise equilibrium for a Stackelberg game. Table 1 captures the strategic interaction between manufacturers and retailers in the supply chain. In the bimatrix, rows define manufacturers' strategies and columns define retailers' strategies.

Table 1: Payoff matrix

M \ R	t	h
T	$\frac{(a-c)^2(1-k_m)(1-k_m k_r)}{2(1-k_r)[2-k_m(1+k_r)]^2}, \frac{(a-c)^2(1-k_m k_r)[1-(3k_m)k_r+k_m {k_r}^2]}{4(1-k_r)[2-k_m(1+k_r)]^2}$	$\frac{(a-c)^2(1-k_m)^2}{2(2-k_m)^2}, \frac{(a-c)^2}{4(2-k_m)^2}$
H	$\frac{(a-c)^2}{8(1-k_r)}, \frac{(a-c)^2(1-3k_r)}{16(1-k_r)}$	$\frac{(a-c)^2}{8}, \frac{(a-c)^2}{16}$

We use the symmetric stakeholder strategy to illustrate how profits presented in Table 1 are derived. From Equation (3), the retailer's best reply function when wholesale price is w:

$$p(w) = \frac{a + k_r c - w(1 - k_r)}{2}$$

The equilibrium prices of the manufacturer and the retailer, respectively, are:

$$w^{Tt} = \frac{a + c - 2ck_r - k_m(a - ck_r^2)}{(1 - k_r)[2 - k_m(1 + k_r)]}, p^{Tt} = \frac{3a + c - k_m(2a + (a + c)k_r)}{2[2 - k_m(1 + k_r)]}$$

The profits of manufacturers and retailers, respectively, are:

$$\pi_m^{Tt} = \frac{(a - c)^2(1 - k_m)(1 - k_m k_r)}{2(1 - k_r))[2 - k_m(1 + k_r)]^2}, \pi_r^{Tt} = \frac{(a - c)^2(1 - k_m k_r))[1 - (3 - k_m)k_r + k_m k_r^2]}{4(1 - k_r)[2 - k_m(1 + k_r)]^2}$$

Profits for other strategy profiles can be derived similarly.

Since k_m and k_r are common knowledge, every firm knows how much their partner in the supply chain values their own profit. From Equations (2) and (3) we analyze the effect of stakeholder's strategy choice on the firm's price.

$$\frac{\partial w^{Tt}}{\partial k_r} = \frac{2(a - c)(1 - k_m)(1 - k_m k_r)}{(1 - k_r)^2[2 - k_m(1 + k_r)]^2} > 0, \frac{\partial p^{Tt}}{\partial k_m} = -\frac{(a - c)(1 - k_r)}{2[2 - k_m(1 + k_r)]^2} < 0,$$

for all $k_m \neq 1$ and $k_r \neq 1$.

For any given level of k_r, the manufacturer's profit rises as w increases. Thus, retailers' concern for its stakeholders (manufacturers) induces manufacturers to use a higher wholesale price. For any given level of k_m, a retailer can increase his profit by selling more units which can be achieved by reducing p. Consequently, manufacturers' concern for its stakeholder induces retailers to use a lower marketing price. Moreover, each firm's profit is increasing in the weight attached by its stakeholder (k_m or k_r) to that firm's profit.

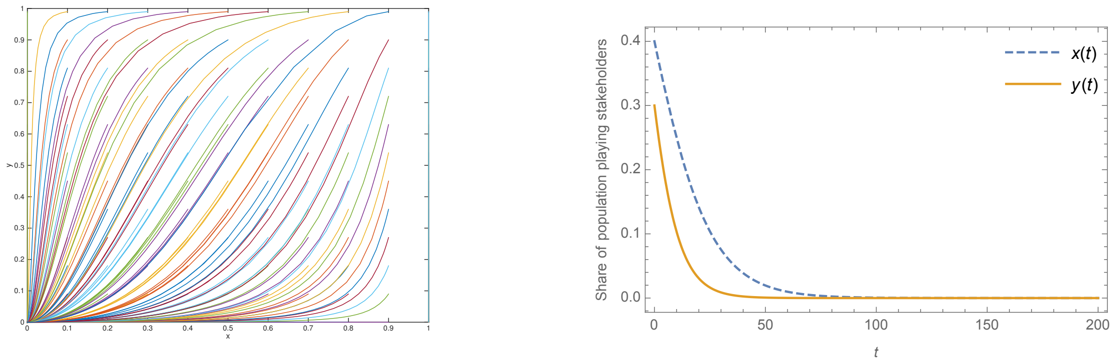
4 Random matching

Consider the following parameter values for the profit functions in Table 1: $a = 4, c = 1, k_m = 0.4, k_r = 0.1$. The payoff matrix for these parameter values is:

$$\mathbf{N} = \begin{bmatrix} (1.18, 0.73) & (1.05, 0.88) \\ (1.25, 0.44) & (1.23, 0.56) \end{bmatrix}$$

This numerical example illustrates that the symmetric shareholder (H, h) strategy is the strong Nash equilibrium in a game of random matching, and this strategy profile is evolutionarily stable (Weibull, 1995). This result persists for other parameter values.

We continue to use the parameter vales presented in bimatrix **N** to analyze evolutionary dynamics of this game under the assumption that population growth follows replicator dynamics. Hence, growth rate of a strategy share in either population is proportional to the difference between the average payoff to that strategy and the average payoff across both strategies in that population. Suppose that a proportion x of manufacturers and a proportion y of retailers are stakeholder-oriented firms. Using the eigenvalue method, we derive that the evolutionarily stable equilibrium occurs when both manufacturer and retailer populations choose shareholder strategy, and that the other strategy profiles are unstable.



(a) Dynamics (b) Share playing T/t

Figure 1: Random matching

Fig. 1 represents the dynamics of the replicator equations. In Fig.1a, X-axis is the measure of stakeholder-oriented manufacturers, and Y-axis is the measure of stakeholder-oriented retailers. The analytic solution shows that the point (0, 0), i.e. both manufacturer and retailer choose to be shareholder-oriented, is the global attractor, and all states converge to this equilibrium (Fig. 1a). As presented in Fig. 1b, as $t \to \infty$, both the share of manufacturers and retailers choosing shareholder strategy tends to 0 ($x \to 0, y \to 0$). Thus, the evolutionarily stable strategy for both populations is to be shareholder-oriented in a game of random matching. This result is presented in Proposition 1.

Proposition 1 *The point (0, 0) is locally asymptotically stable (ESS). The point (0,* *1), (1, 0) and (1, 1) are unstable equilibria. Specially, the point (1, 1) is a source, whereas (0, 1), (1, 0) are saddle points.*

From the profits of the supply chain, we have $\pi_m^{Tt} + \pi_r^{Tt} > \pi_m^{Hh} + \pi_r^{Hh}$. If social benefit is modeled simply as the sum of manufacturer and retailer profits, then social benefit is maximized when both the manufacturer and the retailer pursue stakeholder strategy. However, it is in each agent's own interest to pursue shareholder strategy. Thus, a prisoners' dilemma emerges in a game of random matching.

5 Assortative matching

In reality, most matching is not random; a firm chooses to associate with other firms based on its characteristics. In this section, we consider the more realistic case of non-random matching; specifically, we analyse a game of assortative matching.

5.1 Equilibrium Analysis

Bergstrom (2003) defines the index of assortativity for a one-population, two-strategy game as the difference in probability of an agent encountering another agent of one's own type and that of encountering an agent of the other type. We extend this concept to a two-population game by considering a game where agents' types determine probability of encountering an agent of a specific type in the supply chain. Let $s(x, y)$ be the probability of a T-manufacturer encountering a t-retailer and $q(x, y)$ be the probability of an H-manufacturer encountering a t-retailer. Similarly, we define $u(x, y)$ to be the probability of a t-retailer encountering a T-manufacturer and $v(x, y)$ to be the probability of an h-retailer encountering a T-manufacturer.

We define the index of assortativity for our two-population, two-strategy game as the difference in probability of a particular type of firm (either shareholder-oriented or stakeholder-oriented) matching with a firm of the same type and the probability of matching with a firm of the other type in the supply chain. We consider how strategy choices are affected by the probabilities of encountering a manufacturer or retailer of a specific type through a game of assortative matching. In the following analysis, we suppress arguments of the probability functions to simplify notation.

For a T-manufacturer, the difference in probability between encountering a t-retailer and an h-retailer is

$$\beta_1 = u(x, y) - v(x, y)$$

For an H-manufacturer, the difference in probability between encountering an h-retailer and a t-retailer is

$$\beta_2 = 1 - v(x, y) - [1 - u(xy,)] = u(x, y) - v(x, y)$$

Thus, for both types of manufacturers, the difference in the probability of encountering a retailer of one's own type and a retailer of the other type are identical. We write $\beta_1 = \beta_2 = \beta$.

In the retailer population, the difference between the probability of a t-retailer encountering a T-manufacturer and encountering an H-manufacturer is given by $\alpha_1 = s(x, y) - q(x, y)$. The difference in probability of an h-retailer encountering an H-manufacturer and a T-manufacturer is given by $\alpha_2 = 1 - q(x, y) - [1 - s(x, y)] = s(x, y) - q(x, y)$. Again, for both types of retailers, the difference in probability of encountering a manufacturer of one's own type and a manufacturer of the other type are identical. We write $\alpha_1 = \alpha_2 = \alpha$.

Thus, within each population, the difference in probability of encountering an agent of one's own type and an agent of the other type in the supply chain is equivalent. This result is consistent with the one-population, two-strategy game in Bergstrom (2003) where the difference between the probability of two agents of the same type encountering each other and the probability of two different types matching with each other is equivalent across both strategy-choices in the population. In the rest of our analysis we define $\beta = u(x, y) - v(x, y)$ as the index of assortativity for manufacturers, and $\alpha = s(x, y) - q(x, y)$ as the index of assortativity for retailers.

In the population of N manufacturers and N retailers, firms are matched assortatively to form one-manufacturer, one-retailer supply chains. We characterise the number of supply chains in which a T-manufacturer (H-manufacturer) meets a t-retailer as $N_{Tt}(N_{Ht})$, and the number of supply chains in which a T-manufacturer (H-manufacturer) meets an h-retailers as $N_{Th}(N_{Hh})$. Similarly, we define the number of supply chains in which a t-retailer (h-retailer) meets a T-manufacturer as $N_{tT}(N_{hT})$, and the number of supply chains in which a t-retailer (h-retailer) meets an H-manufacturer as $N_{tH}(N_{hH})$.

The number of supply chains of T-t pairings (N_{Tt} or N_{tT}) formed in a population of size N is the probability that a T-manufacturer meets with a t-retailer ($s(x, y)$) times the number of T-manufacturers ($x \cdot N$), so $N_{Tt} = N \cdot x \cdot s(x, y)$. Likewise, the number of supply chains in which a t-retailer encounters a T-manufacturer is $N_{Tt} = N \cdot y \cdot u(x, y)$.

Clearly, it must be the case that $N_{Tt}=N_{tT}$, so we have $x \cdot s(x,y) = y \cdot u(x,y)$. Similarly, it must be the case that $N_{Th}=N_{hT}$, $N_{Ht}=N_{tH}$, $N_{Hh}=N_{hH}$. Therefore, the following equations hold.

$$\begin{aligned} x \cdot s(x,y) &= y \cdot u(x,y), \\ x[1-s(x,y)] &= [1-y]v(x,y), \\ [1-x]q(x,y) &= y[1-u(x,y)], \\ [1-x][1-q(x,y)] &= [1-y][1-v(x,y)]. \end{aligned}$$

The probabilities of encounter can thus be written as,

$$\begin{aligned} s(x,y) &= y + \alpha[1-x], \\ q(x,y) &= y - \alpha x, \\ u(x,y) &= x + \beta[1-y], \\ v(x,y) &= x - y\beta. \end{aligned}$$

Next, we use these probabilities to derive Nash equilibria of the game under replicator dynamics.

5.2 Replicator dynamics

We continue to assume that growth in strategy-shares within each population is determined by replicator dynamics. For the probability of encounters given in the previous section, average fitness functions among manufacturers and retailers are given by:

$$\begin{aligned} w_m^T &= s(x,y)\pi_m^{Tt} + [1-s(x,y)]\pi_m^{Th}, \\ w_m^H &= q(x,y)\pi_m^{Hh} + [1-q(x,y)]\pi_m^{Hh}, \\ w_r^t &= u(x,y)\pi_r^{Tt} + [1-u(x,y)]\pi_r^{Ht}, \\ w_r^h &= v(x,y)\pi_r^{Th} + [1-v(x,y)]\pi_r^{Hh}. \end{aligned}$$

The payoff advantage functions for T-manufacturers and t-retailers are, respectively,

$$\Delta w_m = \tfrac{(a-c)^2}{8}[1 - y + \alpha x + \tfrac{y-x\alpha}{1-x} - \tfrac{4(1-k_m)(1+\alpha x-y-\alpha)}{(2-k_m)^2} - \tfrac{4(1-k_m)(1-k_m k_r)(y+\alpha-\alpha x)}{(1-k_r)(2-k_m-k_m k_r)^2}],$$

$$\Delta w_r = \tfrac{(a-c)^2}{16}[x - 1 - y\beta - \tfrac{4(x-y\beta)}{(2-k_m)^2} - \tfrac{(3x-1)(x+\beta-y\beta-1)}{x-1} + \tfrac{4[1+(k_m-3)k_r+k_m k_r^2](1-k_m k_r)(y+\alpha-\alpha x)}{(1-k_r)(2-k_m-k_m k_r)^2}].$$

At Nash equilibrium, population shares playing each strategy remain constant. The proportions adopting the various possible strategies remain unchanged in a population when no agent can do any better by switching to another strategy, i.e., it has to be the case that the payoff advantage to either strategy is 0 (so, $\Delta w_m = 0$, $\Delta w_r = 0$). Consequently, equilibrium share of manufacturers and retailers choosing to be shareholder-oriented is given by,

$$x^* = \frac{(\pi_r^{Hh} - \pi_r^{Ht} - \pi_r^{Th} + \pi_r^{Tt})}{(\pi_m^{Hh} - \pi_m^{Ht} - \pi_m^{Th} + \pi_m^{Tt})(\pi_r^{Hh} - \pi_r^{Ht} - \pi_r^{Th} + \pi_r^{Tt})(1-\alpha\beta)} \times \\ [\pi_m^{Hh} - \pi_m^{Th}(1-\alpha) - \pi_m^{Tt}\alpha)\beta + (\pi_m^{Hh} - \pi_m^{Ht} - \pi_m^{Th} - \pi_m^{Tt})(\pi_r^{Ht} - \pi_r^{Hh} - \pi_r^{Ht}\beta + \pi_r^{Tt}\beta)],$$

$$y^* = [(1-\alpha\beta)(\pi_m^{Hh}\pi_r^{Tt} + \pi_m^{Th}\pi_r^{Ht} - \pi_m^{Hh}\pi_r^{Ht} - \pi_m^{Th}\pi_r^{Tt}) + (1+\alpha)\pi_m^{Hh}\pi_r^{Hh} + (1-\alpha)\pi_m^{Th}\pi_r^{Th} - \pi_m^{Hh}\pi_r^{Th} \\ - \pi_m^{Th}\pi_r^{Hh} + \alpha(1-\beta)\pi_m^{Ht}\pi_r^{Ht} + \alpha(1+\beta)\pi_m^{Tt}\pi_r^{Tt} + \alpha(\pi_m^{Tt}\pi_r^{Th} + \pi_m^{Th}\pi_r^{Tt} - \pi_m^{Ht}\pi_r^{Hh} - \pi_m^{Hh}\pi_r^{Ht}) \\ + \alpha\beta(\pi_m^{Tt}\pi_r^{Ht} + \pi_m^{Ht}\pi_r^{Tt})]/[(\pi_m^{Hh} + \pi_m^{Tt} - \pi_m^{Ht} - \pi_m^{Th})(\pi_r^{Hh} + \pi_r^{Tt} - \pi_r^{Ht} - \pi_r^{Th})(1-\alpha\beta)].$$

The continuous replicator dynamics of this game can be represented as,

$$\dot{x} = x(1-x)\Delta w_m = x(1-x)\tfrac{(a-c)^2}{8}[1 - y + \alpha x + \tfrac{y-x\alpha}{1-x} - \tfrac{4(1-k_m)(1+\alpha x-y-\alpha)}{(2-k_m)^2} \\ - \tfrac{4(1-k_m)(1-k_m k_r)(y+\alpha-\alpha x)}{(1-k_r)(2-k_m-k_m k_r)^2}],$$

$$\dot{y} = y(1-y)\Delta w_r = y(1-y)\tfrac{(a-c)^2}{16}[x - 1 - y\beta - \tfrac{4(x-y\beta)}{(2-k_m)^2} - \tfrac{(3x-1)(x+\beta-y\beta-1)}{x-1} \\ + \tfrac{4[1+(k_m-3)k_r+k_m k_r^2](1-k_m k_r)(x+\beta-y\beta}{(1-k_r)(2-k_m-k_m k_r)^2}.$$

We revert to the previous numerical example to contrast equilibria of the games of random and assortative matching. In addition to the parameter values assumed in Section 4, let the indices of assortativity be $\alpha = 0.6$, $\beta = 0.5$. For these parameter values, the payoff difference for a T- manufacturer and t-retailer are, respectively:

$$\Delta w_m = w_m^T - w_m^H = 0.007 - 0.002x + 0.003y, \\ \Delta w_r = w_r^t - w_r^h = 0.023 - 0.02x + 0.01y.$$

The replicator equations are:

$$\dot{x} = x(1-x)(0.007 - 0.002x + 0.003y), \quad (4)$$

$$\dot{y} = y(1-y)(0.023 - 0.02x + 0.01y). \quad (5)$$

Phase portrait of the replicator System (4-5) is presented in Fig. 2. The point (1, 1), where both manufacturers and retailers choose stakeholder strategy, is the global attractor and all agents converge to this equilibrium. However, equilibrium strategy profile is determined by parameter values. Thus, unlike a game of random matching, the game of assortative matching does not support always dominant strategy.

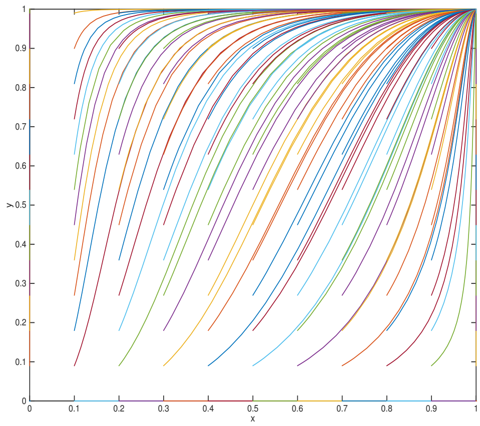


Figure 2: Phase portrait of System (4-5)

Proposition 2 summarizes the evolutionary stable strategies supported by a game of assortative matching.

Proposition 2 *(1) When* $\beta < \frac{2k_rC}{E(1-k_r)^2}$ *and* $\alpha < \frac{k_m^2(1-k_r)C}{4k_r(1-k_m)A}$*, the point (0, 0) is asymptotically stable (ESS);*
(2) when $\alpha > \frac{k_m^2(1-k_r)}{k_r(2-k_m)^2}$ *and* $\beta < \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_r)^2}$*, the point (1, 0) is asymptotically stable (ESS);*
(3) when $\beta > \frac{2k_r(2-k_m)^2}{k_m(4-k_m)(1-k_r)}$ *and* $\alpha < \frac{k_m^2(2-k_m)^2(1-k_r)^2}{4k_r(1-k_m)A}$*, the point (0, 1) is asymptotically stable (ESS);*
(4) when $\alpha > \frac{k_m^2(1-k_r)^2}{Ck_r}$ *and* $\beta > \frac{4k_r(1-k_m)B}{Ck_m(4-k_m)(1-k_r)}$*, the point (1, 1) is asymptotically stable (ESS).*

Here,

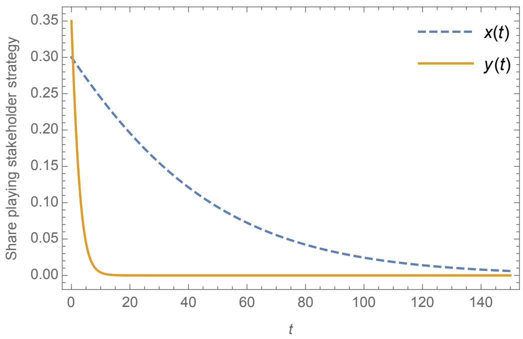
$$A = 4 - 4k_m + 3k_m^2 - k_m^3 - 4k_mk_r + k_m^2k_r + k_m^2k_r^2 > 0,$$
$$B = 8 - 4k_m - 12k_mk_r + 7k_m^2k_r - k_m^3k_r + 3k_m^2k_r^2 - k_m^3k_r^2 > 0,$$
$$C = (2 - k_m - k_mk_r)^2 > 0,$$
$$E = k_m(4 - k_m - k_mk_r) > 0.$$

Proof is presented in Appendix 2.

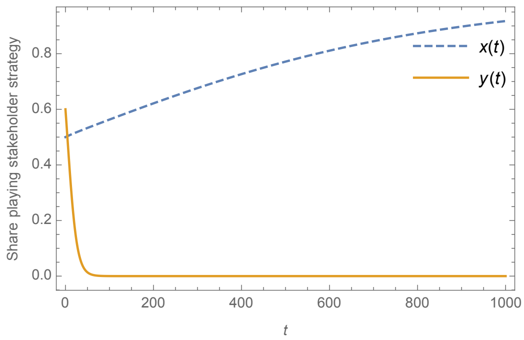
For relatively small values of the index of assortativity, the probability of encountering an agent of one's own type and of the other type are similar. Our analysis shows that both types of firms choose to be shareholder-oriented when the index of assortativity is small and positive (Proposition 2(1)). Thus, though firms benefit more from a Tt match than an Hh match, the relatively comparable probabilities of matching with a firm of one's own type and a firm of the other type deters firms from being stakeholder-oriented. This result is presented in Fig. 3a, where the parameter values are $k_m = 0.2, k_r = 0.3, \alpha = -0.03, \beta = 0.03$. With these low values of the indices of assortativity, both manufacturers and retailers choose to be shareholder-oriented in the long run.

The asymmetric strategy profile in which the manufacturer chooses shareholder-oriented (stakeholder-oriented) strategy and the retailer chooses stakeholder-oriented strategy (shareholder-oriented) is evolutionarily stable if the index of assortativity for manufacturer is relatively low (relatively high), and the index of assortativity for retailer is relatively high (relatively low). When the manufactures' index of assortativity is relatively low, the manufacturer's game is comparable to the case of random matching. With a relatively high index of assortativity, retailers are more likely to be matched with a manufacturer of their own type than of the other type. The payoff functions show that the retailer benefits more from a symmetric match than an asymmetric match. Thus, with a relatively high index of assortativity, retailers choose to be stakeholder-oriented. Figs. 3b and 3c show evolutionarily stable asymmetric strategy profile when parameter values fulfill conditions in Proposition 2(2) and 2(3). The parameter values we use in these figures are $k_m = 0.02, k_r = 0.1, \alpha = 0.13, \beta = 0.3$ and $k_m = 0.5, k_r = 0.1, \alpha = 0.35, \beta = 0.3$, respectively.

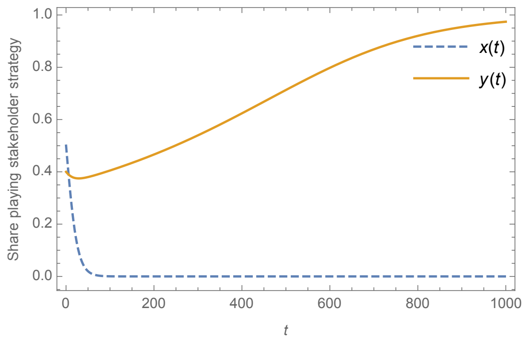
When the probability of matching with one's own type is higher than the probability of matching with the other type (Proposition 2(4)), firms choose to be stakeholder-oriented. This result is presented in Fig. 3d, where the values of parameters are $k_m = 0.4, k_r = 0.1, \alpha = 0.6, \beta = 0.5$. In our analysis, $a = 4$ and $c = 1$; however, the dynamics of the game do not depend on the values of a and c.



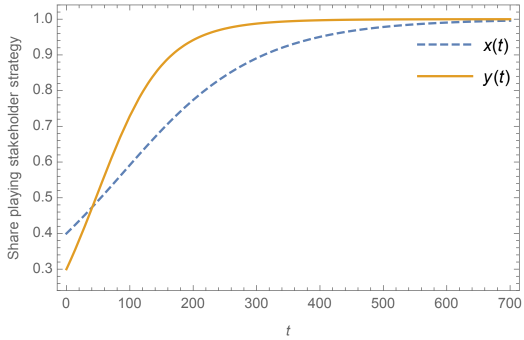
(a) Trajectory for Sys. (4-5) under Case (1)



(b) Trajectory for Sys. (4-5) under Case (2)



(c) Trajectory for Sys. (4-5) under Case (3)



(d) Trajectory for Sys. (4-5) under Case (4)

Figure 3: Share playing T/t

In reality, we see that market equilibrium often support multiple types of matches between firms. For instance, some markets may reach an equilibrium that supports two evolutionarily stable matches, with some supply chains being comprised entirely of shareholder-oriented firms and others being comprised of shareholder-oriented upstream firms and stakeholder-oriented downstream; alternatively, a market could reach an evolutionarily stable equilibrium with some wholly shareholder-oriented supply chains and other supply chains with stakeholder-oriented upstream firms and shareholder-oriented downstream firms. What would cause markets to evolve into one of these and not the other type of market? While a detailed analysis of this question is beyond the scope of the current paper, our succinct explanation is that initial conditions matter. These conditions include political, historical, or environmental factors, among many others. We use initial parameter values to describe aggregate initial conditions. In Corollary 1, we describe this simplified explanation by showing that for various initial parameter values, a game of assortative matching can support multiple types of supply chains in equilibrium.

Corollary 1 *(1) When* $\frac{k_m^2(1-k_r)}{k_r(2-k_m)^2} < \alpha < \frac{k_m^2(1-k_r)C}{4k_r(1-k_m)A}$ *and* $\beta < min\{\frac{2k_rC}{E(1-k_r)^2}\}$*, both (0, 0) and (1, 0) are ESS;*

(2) when $\frac{2k_r(2-k_m)^2}{k_m(4-k_m)(1-k_r)} < \beta < \frac{2k_rC}{E(1-k_r)^2}$ *and* $\alpha < min\{\frac{k_m^2(1-k_r)C}{4k_r(1-k_m)A}, \frac{k_m^2(2-k_m)^2(1-k_r)^2}{4k_r(1-k_m)A}\}$*, both (0,0) and (0,1) are ESS;*

(3) when $\frac{k_m^2(1-k_r)}{k_r(2-k_m)^2} < \alpha < \frac{k_m^2(2-k_m)^2(1-k_r)^2}{4k_r(1-k_m)A}$ *and* $\frac{2k_r(2-k_m)^2}{k_m(4-k_m)(1-k_r)} < \beta < \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_r)}$*, both (1,0) and (0,1) are ESS;*

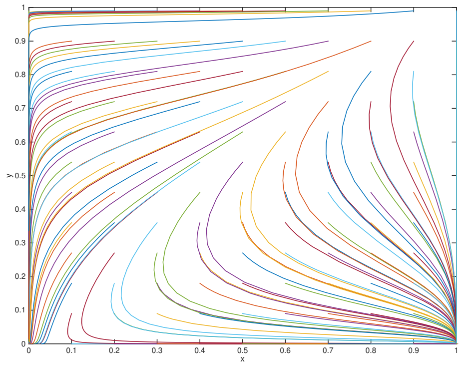
(4) when $\frac{4k_r(1-k_m)B}{Ck_m(4-k_m)(1-k_r)} < \beta < \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_r)^2}$ *and* $\alpha > max\{\frac{k_m^2(1-k_r)^2}{Ck_r}, \frac{k_m^2(1-k_r)}{k_r(2-k_m)^2}\}$*, both (1,0) and (1,1) are ESS;*

(5) when $\beta > max\{\frac{4k_r(1-k_m)B}{Ck_m(4-k_m)(1-k_r)}, \frac{2k_r(2-k_m)^2}{k_m(4-k_m)(1-k_r)}\}$ *and* $\frac{k_m^2(1-k_r)^2}{Ck_r} < \alpha < \frac{k_m^2(2-k_m)^2(1-k_r)^2}{4k_r(1-k_m)A}$*, both (0,1) and (1,1) are ESS;*

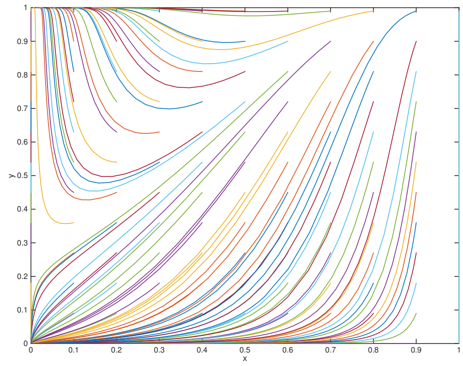
(6) when $\frac{k_m^2(1-k_r)}{k_r(2-k_m)^2} < \alpha < min\frac{k_m^2(1-k_r)C)}{4k_r(1-k_m)A}, \frac{k_m^2(2-k_m)^2(1-k_r)^2}{4k_r(1-k_m)A}$ *and* $\frac{2k_r(2-k_m)^2}{k_m(4-k_m)(1-k_r)} < \beta < min\{\frac{2k_rC}{E(1-k_r)^2}, \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_r)^2}\}$*, (0, 0), (0, 1) and (1, 0) are ESS;*

(7) when $max\{\frac{k_m^2(1-k_r)}{k_r(2-k_m)^2}, \frac{k_m^2(1-k_r)^2}{Ck_r}\} < \alpha < \frac{k_m^2(2-k_m)^2(1-k_r)^2}{4k_r(1-k_m)A}$ *and* $max\ \{\frac{4k_r(1-k_m)B}{Ck_m(4-k_m)(1-k_r)}, \frac{2k_r(2-k_m)^2}{k_m(4-k_m)(1-k_r)}\} < \beta < \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_r)^2}$*, (1, 0),(0,1) and (1,1) are ESS.*

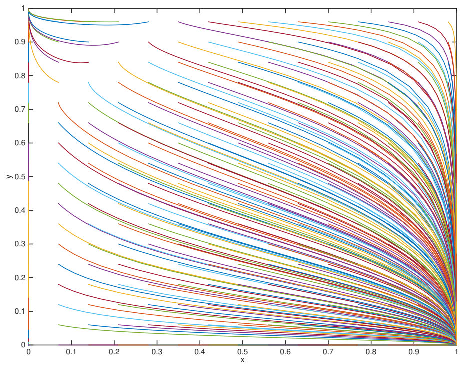
Fig.4 presets the dynamics of games with multiple evolutionarily stable equilibria. Fig 4a shows that for certain initial conditions, manufacturers' index of assortativity determines whether manufacturers are shareholder-oriented or stakeholder-oriented in the evolutionarily stable equilibrium. For the range of initial conditions, identified in Corollary 1(2), retailers may be shareholder-oriented or stakeholder-oriented in the evolutionarily stable equilibrium while all manufacturers are shareholder-oriented, as depicted in Fig.4b. The asymmetric equilibrium derived in Corollary 1(3) is presented in Fig. 4c, where manufacturers and retailers choose opposite strategies in the evolutionarily stable equilibrium. For the initial parameter values in Corollary 1(4), equilibrium is characterized by stakeholder-oriented manufacturers and both types of retailers, whereas Corollary 1(5) characterizes the equilibrium with stakeholder-oriented retailers and both types of manufacturers. Figs. 4f and 4g present equilibria with three evolutionary stable strategy profiles.



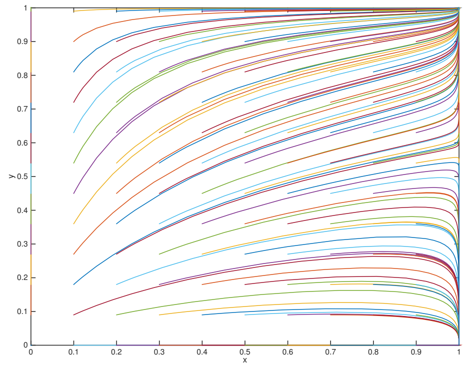
(a) $k_m = .782, k_r = .333, \alpha = .895, \beta = .58$



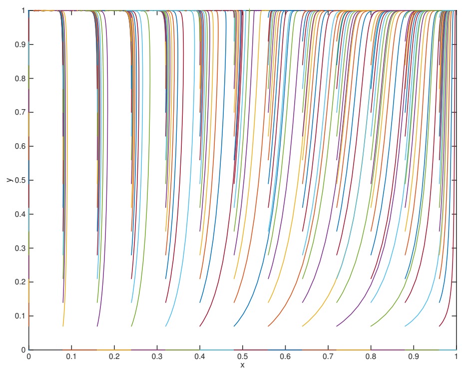
(b) $k_m = .5, k_r = .3, \alpha = .39, \beta = .7$



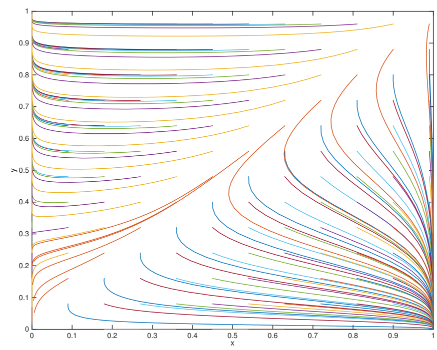
(c) $k_m = .77, k_r = .31, \alpha = .96, \beta = .55$



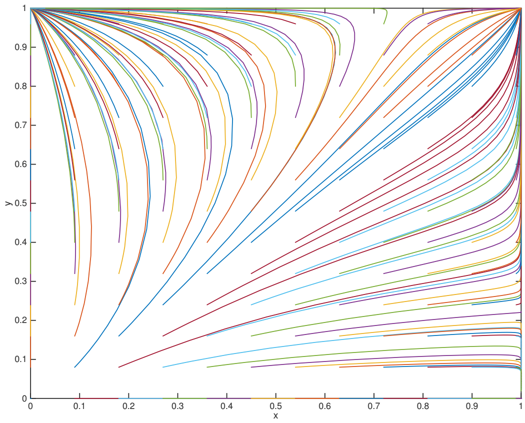
(d) $k_m = .72, k_r = .28, \alpha = .92, \beta = .6$



(e) $k_m = .72, k_r = .28, \alpha = .83, \beta = .62$



(f) $k_m = .782, k_r = .333, \alpha = .883, \beta = .592$



(g) $k_m = .77, k_r = .31, \alpha = .94, \beta = .57$

Figure 4: Phase portrait of Corollary 1

Thus, when compared to a game of random matching, dynamics of the game under assortative matching yields the more realistic result of multiple evolutionarily stable steady states.

Furthermore, we consider the special case where both manufacturers and retailers are equally concerned about their stakeholders' utility, i.e., $k_m = k_r$. We find that retailers always adopt shareholder strategy in the long run and that manufacturers' strategy is determined by their index of assortativity.

Corollary 2 *When $0 \leq k_m = k_r = k < 1/3$, we have the following results:*
(1) If $\alpha < \hat{\alpha}_1 = \frac{k(1-k)(2+k)^2}{4(4-k^2-k^3)}$, (0,0) is an ESS;
(2) if $\alpha > \hat{\alpha}_2 = \frac{k(1-k)}{(2-k)^2}$, (1,0) is an ESS.

According to Corollary 2, if both the manufacturer and the retailer care about their stakeholder moderately and with comparable magnitudes ($0 \leq k_m = k_r = k < 1/3$), the retailer adopts shareholder strategy in the long term. If the index of assortativity for retailer is small ($\alpha < \hat{\alpha}_1$), the strategy profile where both manufacturers and retailers choose shareholder strategy is evolutionarily stable; when the index of assortativity for retailers is relatively high ($\alpha > \hat{\alpha}_2$), manufacturers chooses to be stakeholder-oriented and retailers choose to be shareholder-oriented in the long run.[2] Furthermore, we find that the degree to which manufacturers and retailers care about their stakeholder

[2]It follows as a corollary of Proposition 2 that no ESS exists when $\hat{\alpha}_1 < \alpha < \hat{\alpha}_2$.

increases the critical value at which the manufacturer population switches over to stakeholder strategy. As k increases, the value α at which manufacturers switch from stakeholder strategy to shareholder strategy increases.

In this game, a match is positive assortative when the symmetric strategy is ESS; the match is negative assortative when the asymmetric strategy is ESS. We use these match-types to study how the indices of assortativity affect a game of assortative matching.

Proposition 3 *The indices of assortativity for manufacturers and retailers determine whether the match is positive assortative or negative assortative; specifically,*
(1) when $\alpha > \frac{k_m^2(1-k_r)}{k_r(2-k_m)^2}$ *and* $\beta < \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_m)^2}$, *or* $\beta > \frac{2k_r(2-k_m)^2}{k_m(4-k_m)(1-k_r)}$ *and* $\alpha < \frac{k_m^2(2-k_m)^2(1-k_r)^2}{4k_r(1-k_m)A}$, *the match is negative assortative;*
(2) when $\alpha < \frac{k_m^2(1-k_r)C}{4k_r(1-k_m)A}$ *and* $\beta < \frac{2k_rC}{E(1-k_r)^2}$, *or* $\alpha > \frac{k_m^2(1-k_r)^2}{Ck_r}$ *and* $\beta > \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_m)^2}$, *the match is positive assortative.*

Proposition 3 provides the sufficient conditions for positive assortative and negative assortative matching. Positive assortative matching occurs only when the indices of assortativity for both the manufacturer and the retailer populations are concurrently high or low enough. In other words, manufacturers and retailers of the same type are more likely to match with each other when the probability of encountering one's own type is high or low enough for both populations. The match is negative assortative when the manufacturer's (retailer's) index of assortativity is high and the index of assortativity for retailer (manufacturer) is moderate or low.

5.3 Evolutionary stability of mixed strategy

In this subsection, we analyse stability of interior equilibria. Complete analysis is intractable but numerical simulations show that the interior equilibria are unstable. Let $k_m = 0.45$ and $k_r = 0.2$. Consider a range of α and β such that $(x^*, y^*) \in (0,1) \times (0,1)$ as shown in Fig. 5. The range of α and β such that the real part of the eigenvalue of the Jacobian matrix at the point (x^*, y^*) is less than or equal to zero is shown in the Fig. 6.

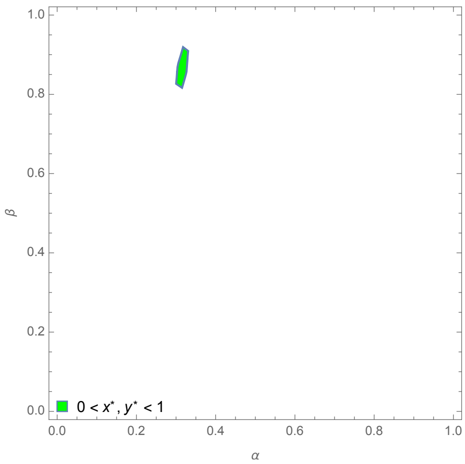


Figure 5: Range of parameters such that $(x^*, y^*) \in (0, 1) \times (0, 1)$.

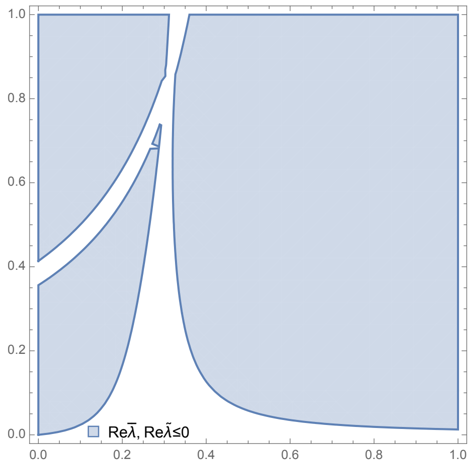


Figure 6: Range of parameters such that that real part of the eigenvalue $(\bar{\lambda}, \tilde{\lambda})$ of the Jacobian matrix at (x^*, y^*) is less than or equal to zero.

We combine Figs. 5 and 6 to show that the interior equilibrium is not asymptotically stable (Fig. 7). We have checked these results with other parameter values and they are broadly consistent with what is presented in Figs. 5 and 6 (see Appendix 3). Thus, assortative matching does not support a stable mixed equilibrium.

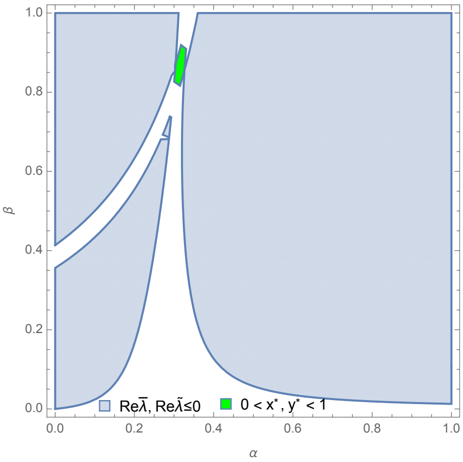


Figure 7: Assortative matching does not support asymptotically stable interior equilibrium

6 Conclusion

In this paper, we use games of random and assortative matching to consider the strategic decisions of firms to be shareholder-oriented or stakeholder-oriented in single-manufacturer, single-retailer supply chains. In our basic model, we derive the wholesale price of the manufacturer and the market price of the retailer in a one-shot game. We show that the more retailers care about their stakeholders, the higher is manufacturers' wholesale price, whereas the more manufacturers care about their stakeholders, the lowers is retailers' selling price. A firm's profit increases with its stakeholder's concern for that firm. A game of random matching does not support an interior equilibrium and the unique evolutionarily stable equilibrium occurs when both manufacturer and retailer choose shareholder strategy. This is also the dominant strategy for manufacturer and retailer.

With assortative matching, the game has an interior Nash equilibrium. However, this equilibrium is unstable. All four strategy profiles considered in this paper may be evolutionarily stable for different values of the indices of assortativity. The symmetric shareholder strategy is evolutionarily stable when the indices of assortativity for both the retailer and the manufacturer are low. When the index of assortativity for manufacturer is high (moderate), and the index of assortativity for retailer is moderate (high), the asymmetric strategy profiles are evolutionarily stable, and matching is negative assortative. The symmetric stakeholder strategy profile is evolutionary stable and the match is positive assortative when the index of assortativity for both manufacturers and retailers is concurrently high (or low) enough.

Our paper shows that the dynamics of the game is more variable under assortative matching than random matching. We want to extend this research by considering games in which the probability of assortative matching is a specific function of the share of stakeholder-oriented manufacturers and retailers. Furthermore, we are also interested in analysing the effect of stakeholder-oriented strategy on double marginalization in the channel. We would also like to incorporate uncertainty into our model in the future. This is a more realistic setting and it would be interesting to study the impact of uncertainty, such as demand shocks or cost shocks, on the stability of equilibrium points.

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A Appendix

A.1 Proof of Proposition 1

The replicator equation is:

$$\dot{x} = x(1-x)[y\pi_m^{Tt} + (1-y)\pi_m^{Th} - y\pi_m^{Ht} - (1-y)\pi_m^{Hh}] = F(x,y), \tag{A.1}$$

$$\dot{y} = y(1-y)[x\pi_r^{Tt} + (1-x)\pi_r^{Ht} - x\pi_r^{Th} - (1-x)\pi_r^{Hh}] = G(x,y). \tag{A.2}$$

Equilibria obtained by solving Equations (A.1) and (A.2) are (0, 0), (0, 1), (1, 0) and (1, 1). There are no interior points of equilibrium. We use the Jacobin matrix of the replicator equation to analyse the stability of these equilibrium points.

$$J = \begin{bmatrix} \dfrac{\partial F(x,y)}{\partial x} & \dfrac{\partial F(x,y)}{\partial y} \\ \dfrac{\partial G(x,y)}{\partial x} & \dfrac{\partial G(x,y)}{\partial y} \end{bmatrix} =$$

$$\begin{bmatrix} (2x-1)(y\pi_m^{Tt} + (1-y)\pi_m^{Tt} - y\pi_m^{Ht} - (1-y)\pi_m^{Hh} & x(1-x)(\pi_m^{Tt} - \pi_m^{Th} - \pi_m^{Ht} + \pi_m^{Hh}) \\ y(1-y)(\pi_r^{Tt} - \pi_r^{Ht} - \pi_r^{Th} + \pi_r^{Hh}) & (2y-1)(x\pi_r^{Tt} + (1-x)\pi_r^{Ht} - x\pi_r^{Th} - (1-x)\pi_r^{Hh}) \end{bmatrix}$$

Hence, the eigenvalues of the Jacobian matrix **J** at the point (0, 1) are

$$\lambda_1 = -\frac{(a-c)^2 k_m^2 (1-k_r)}{8[2-k_m(1+k_r)]^2},$$

$$\lambda_2 = \frac{(a-c)^2 k_r}{8(1-k_r)}.$$

For $0 \leq k_r \leq 1/3$, we see that $\lambda_1 < 0$ and $\lambda_2 > 0$. Hence (0, 1) is a saddle point. Similarly, we evaluate the eigenvalues of the Jacobian matrix **J** for the other points (1, 0), (1, 1) and (0, 0). Thus, Proposition 1 is true.

A.2 Proof of Proposition 2

Part 1 The eigenvalues of Jacobin matrix **J** at the point (0, 0) are,

$$\lambda_3 = -\frac{(a-c)^2(2k_rC - \beta(1-k_r)^2E)}{16(1-k_r)(2-k_m(1+k_r))^2},$$
$$\lambda_4 = -\frac{(a-c)^2(k_m(1-k_r)C - 4\alpha k_r(1-k_m)A)}{8(2-k_m)^2(1-k_r)(2-k_m(1-k_r))^2}.$$

Here,

$$A = 4 - 4k_m + 3k_m^2 - k_m^3 - 4k_mk_r + k_m^2k_r + k_m^2k_r^2,$$
$$B = 8 - 4k_m - 12k_mk_r + 7k_m^2k_r - k_m^3k_r + 3k_m^2k_r^2 - k_m^3k_r^2,$$
$$C = (2 - k_m - k_mk_r)^2,$$
$$E = k_m(4 - k_m - k_mk_r).$$

Furthermore, $A = 4 - 4k_m + 3k_m^2 - k_m^3 - 4k_mk_r + k_m^2k_r + k_m^2k_r^2 = 2(1-k_m)^2 + 2(1-k_mk_r)^2 + k_m^2 - k_m^3 + k_m^2k_r + k_m^2k_r^2 > 0$, and it can be easily derived that $\beta < \frac{2k_rC}{E(1-k_r)^2}$ and $\alpha < \frac{k_m^2(1-k_r)C}{4k_r(1-k_m)A}$, the eignevalues λ_3 and λ_4 are negative; that is the point (0, 0) is asymptotically stable.

Part 2 The eigenvalues of Jacobin matrix $\mathbf{J}$ at the point (1, 0) are,

$$\lambda_5 = \frac{(a-c)^2[k_m^2(1-k_r) - \alpha k_r(2-k_m)^2]}{8(2-k_m)^2(1-k_r)},$$
$$\lambda_6 = -\frac{(a-c)^2[4(1-k_m)k_rB - \beta(2-k_m)^2(1-k_r)^2E]}{16(2-k_m)(1-k_r)[2-k_m(1+k_r)]^2}.$$

Here,

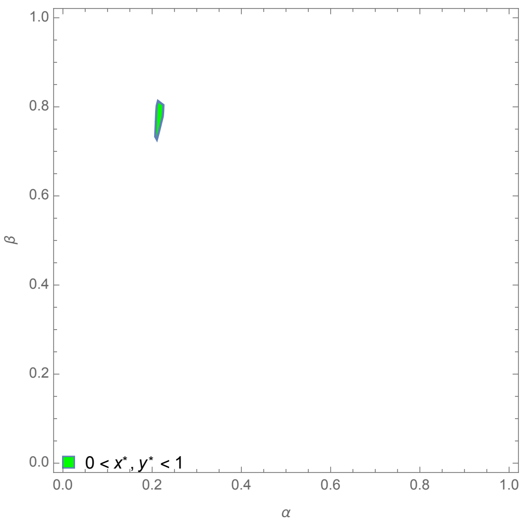
$$\begin{aligned} B &= 8 - 4k_m - 12k_mk_r + 7k_m^2k_r - k_m^3k_r + 3k_m^2k_r^2 - k_m^3k_r^2 \\ &\geq 8 - 4k_m - 12k_mk_r + 6k_m^2k_r + 2k_m^2k_r^2 \\ &= 6(1-k_mk_r)(1-k_m) + 2(1-k_mk_r)^2 + 2k_m(1-k_r) > 0. \end{aligned}$$

We have $\alpha > \frac{k_m^2(1-k_r)}{k_r(2-k_m)^2}$ and $\beta < \frac{4k_r(1-k_m)B}{E(2-k_m)^2(1-k_m)^2}$, the eigenvalues λ_5 and λ_6 are negative, so the point (1, 0) is asymptotically stable.

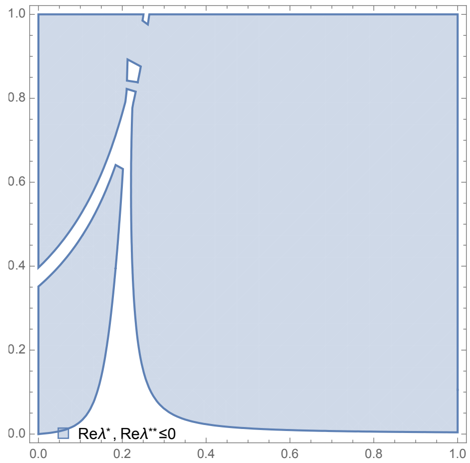
Similarly, we can prove cases (3) and (4) hold. Hence Proposition 2 is true.

A.3 The complement for the analysis of interior equilibrium under assortative matching

Basing on the analysis in the paper, the characteristic of equilibria independence of the values of a and c, so we still fix $a = 4$ and $c = 1$. Here we let $k_m = 0.3$ and $k_r = 0.12$. Figs. 8a and 8b below correspond to Figs. 5 and 6.



(a) Range of parameters such that $(x^*, y^*) \in (0,1) \times (0,1)$.



(b) Range of parameters such that that real part of the eigenvalue $(\bar{\lambda}, \tilde{\lambda})$ of the Jacobian matrix at (x^*, y^*) is less than or equal to zero.

Figure 8: Range of parameters

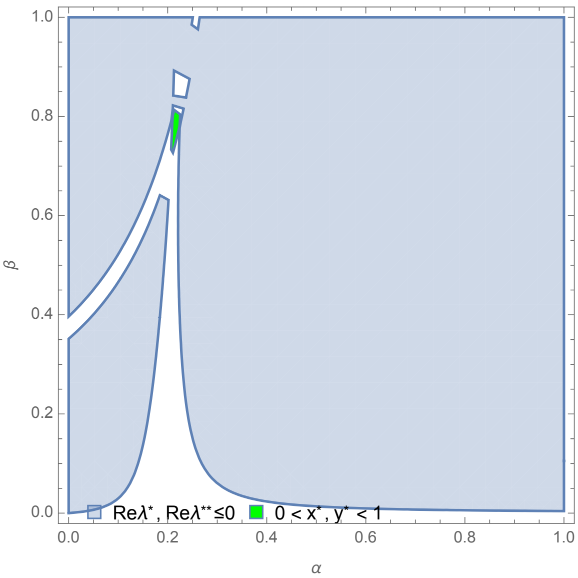


Figure 9: Overlaying Figs. 8a and 8b.

From Fig. 9, we see that there is no overlap in Figs. 8a and 8b. This result is consistent with those in Figs. 5, 6 and 7.